BoltGroup Spreadsheet Algorithm and Verification

This Mathcad worksheet documents the formulas and calculation flow implemented in the **BoltGroup** Excel spreadsheet for determining the shear capacity of a custom bolt group subjected to in-plane forces. It uses both *Elastic and Instantaneous Center of Rotation (IC) methods* from the *AISC Steel Construction Manual, 16th Edition*.

For verification, a standard rectangular 3 × 4 bolt pattern was chosen to compare results with the C-coefficient listed *AISC Table 7-11,* but the process works for any bolt layout..
The formulations *utilize vector operations*, where multiplying two vectors gives a single value equal to the sum of their component products.

Bolt shear resistance $\phi R_n := 18.02 \cdot kip$ A325 N, 3/4" dia.

Bolt coordinates

x (in)	y (in)
−3	−4.5
0	−4.5
3	−4.5
−3	−1.5
0	−1.5
3	−1.5
−3	1.5
0	1.5
3	1.5
−3	4.5
0	4.5
3	4.5

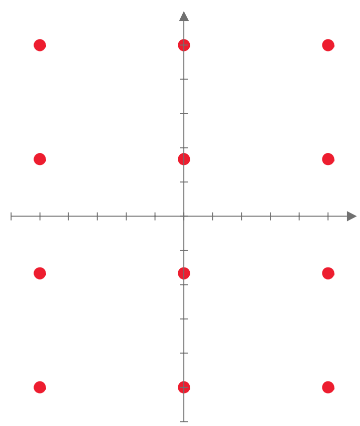
Load Vectors

x_p, y_p - coordinates of application point
β - angle from the x- axis (counterclockwise)
P - intensity (always positive)

Moment in faying surface (+ for counterclockwise):

$M := -400 \cdot kip \cdot in$

x_p (in)	y_p (in)	β (deg)	P (kip)
2	3.44	−120	60
2	−0.88	−90	60



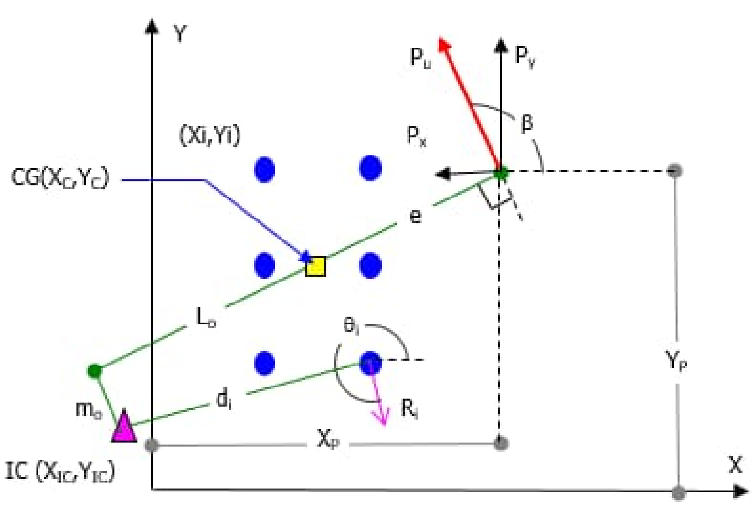


Figure 1 - Geometry

Bolt group properties

Number of bolts: $N_b := \text{rows}(x) = 12$

Bolt group center of gravity (C.G.) $\begin{bmatrix} X_c \\ Y_c \end{bmatrix} := \begin{bmatrix} \text{mean}(x) \\ \text{mean}(y) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ *in*

Polar moment of inertia: $I_p := \sum \left(\left(x - X_c\right)^2 + \left(y - Y_c\right)^2 \right) = 207\ in^2$

Force resultant

Sum the forces by multiplying the vectors:

$P_x := P \cdot \cos(\beta) = -30\ kip$
$P_y := P \cdot \sin(\beta) = -111.96\ kip$
$P_u := \sqrt{P_x^{\ 2} + P_y^{\ 2}} = 115.91\ kip$

Angle of resultant force measured from horizontal axis:
$\beta_o := \text{atan2}\left(P_x, P_y\right) = -105\ deg$

Moment resultant

$M_o := P \cdot \overrightarrow{\left(\sin(\beta) \cdot \left(x_p - X_c\right) - \cos(\beta) \cdot \left(y_p - Y_c\right)\right)} + M$

Adjust angle to keep eccentricity always positive: $\beta_o := \beta_o + \left(M_o < 0\right) \cdot \pi = 75\ deg$
Angle from vertical axis = $90 \cdot deg - \beta_o = 15\ deg$
Eccentricity $e_o := \left| \dfrac{M_o}{P_u} \right| = 4.492\ in$

Origin of resultant force

$X_p := X_c + e_o \cdot \sin\left(\beta_o\right) = 4.339\ in$
$Y_p := Y_c - e_o \cdot \cos\left(\beta_o\right) = -1.163\ in$

Summary of Resultant Force

$P_u = 115.911\ kip$
$\beta_o = 75\ deg$
$e_o = 4.492\ in$
$X_p = 4.339\ in$
$Y_p = -1.163\ in$

Elastic method

Shear forces in the bolts $R_{elastic} := \sqrt{\left(\frac{P_y}{N_b} + \frac{M_o}{I_p} \cdot (x - X_c)\right)^2 + \left(\frac{P_x}{N_b} + \frac{M_o}{I_p} \cdot (y - Y_c)\right)^2}$

$R_{max} := \max(R_{elastic}) = 21.813\ \boldsymbol{kip}$

Bolt group shear resistance $R_{u.elastic} := P_u \cdot \left(\frac{\phi R_n}{R_{max}}\right) = 95.75\ \boldsymbol{kip}$

Method of Instantaneous Center of Rotation (IC).

For geometry definitions see Figure 1 - Geometry.

Define functions to be used in the equation solver:
Define location of IC by two arguments: l and m - distances from the bolt group's center of gravity (C.G.) to the IC, measured normal and parallel to the resultant of acting force (P_u).

Coordinates of IC:
$$x_{IC}(l, m) := -l \cdot \sin(\beta_o) - m \cdot \cos(\beta_o) + X_c$$
$$y_{IC}(l, m) := l \cdot \cos(\beta_o) - m \cdot \sin(\beta_o) + Y_c$$

Direction of shear force in each bolt: $\theta(l, m) := \operatorname{atan2}(x - x_{IC}(l, m), y - y_{IC}(l, m)) - \frac{\pi}{2}$

Distance from IC to each bolt $d(l, m) := \sqrt{(x - x_{IC}(l, m))^2 + (y - y_{IC}(l, m))^2}$

Maximum deformation in the bolt most remote from the IC (per AISC) : $\Delta_{max} := 0.34 \cdot \boldsymbol{in}$
Deformations in the other bolts are proportional to the distance from IC :
$$\Delta(l, m) := \Delta_{max} \cdot \frac{d(l, m)}{\max(d(l, m))}$$
For better conversion while solving a system of equations for in-plane static equilibrium define function for unitless resistance of each bolts defined as a ratio to nominal resistance (R/ϕRn):
$$r(l, m) := \left(1 - \exp\left(-10 \cdot \frac{\Delta(l, m)}{\boldsymbol{in}}\right)\right)^{0.55}$$ (AISC Figure 7-3)

Similarly, define a unitless resistance of the entire bolt group (or coefficient C in AISC tables: $C = \frac{R_{u.ic}}{\phi R_n}$

Solve system of static equilibrium equations

Define initial values for unknown variables and solve equations using Mathcad Solver.

Guess Values

$C := \frac{R_{u.elastic}}{\phi R_n}$ $L_o := 0 \cdot in$ $m_o := 0 \cdot in$

Constraints

$r(L_o, m_o) \cdot cos(\theta(L_o, m_o)) + C \cdot cos(\beta_o) = 0$ Sum(Fx) = 0

$r(L_o, m_o) \cdot sin(\theta(L_o, m_o)) + C \cdot sin(\beta_o) = 0$ Sum(Fy) = 0

$r(L_o, m_o) \cdot \frac{d(L_o, m_o)}{in} - C \cdot \frac{(e_o + L_o)}{in} = 0$ Sum(M) = 0

$L_o \geq 0 \cdot in$

$C \leq N_b$

Solver

$\begin{bmatrix} C \\ L_o \\ m_o \end{bmatrix} := \text{Find}(C, L_o, \mathbf{m_o})$ $\begin{bmatrix} C \\ L_o \cdot in^{-1} \\ m_o \cdot in^{-1} \end{bmatrix} = \begin{bmatrix} 6.957 \\ 3.581 \\ -0.244 \end{bmatrix}$

Summary of Analysis

Coordinates of instantaneous center: $\begin{bmatrix} X_{IC} \\ Y_{IC} \end{bmatrix} := \begin{bmatrix} x_{IC}(L_o, m_o) \\ y_{IC}(L_o, m_o) \end{bmatrix} = \begin{bmatrix} -3.396 \\ 1.162 \end{bmatrix} in$

Bolt group shear resistance

by IC method: $R_{u.ic} := C \cdot \phi R_n = 125.36\ kip$

by Elastic method: $R_{u.elastic} = 95.75\ kip$

Ratio: $\frac{R_{u.ic}}{R_{u.elastic}} = 1.309$

Demand vs Capacity Ratios using both methods:

$DCR := P_u \div \begin{bmatrix} R_{u.ic} \\ R_{u.elastic} \end{bmatrix} = \begin{bmatrix} 0.925 \\ 1.211 \end{bmatrix}$ $\overrightarrow{Check(DCR \leq 1.0)} = \begin{bmatrix} \text{"Good"} \\ \text{"N.G."} \end{bmatrix}$

Compare to the results from AISC Table 7-11 Angle = 15 deg

s = 3", n = 4, $e_x := |X_p| + |Y_p| \cdot \tan(15 \cdot deg) = 4.651\ in$

Interpolate for C values between ex=4" and ex=5"

$C_{AISC} := \frac{(7.55 - 6.67)}{(5 - 4)} \cdot (5 - 4.651) + 6.67 = 6.977$

Compare with $C = 6.957$ $\frac{C_{AISC} - C}{C_{AISC}} = 0.003$ Good

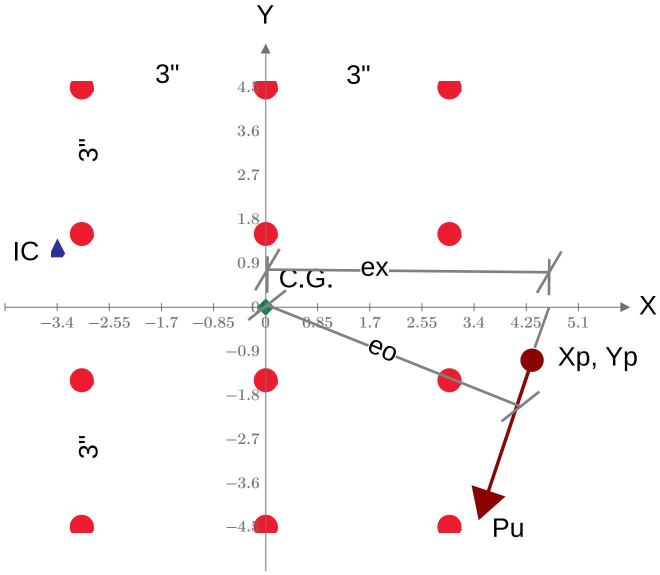


Figure 2 - IC Method Results