readme

This document details the algorithms and formulas used to develop the SectProp.xlsb Excel spreadsheet, available for download at www.yakpol.net.

These calculations are presented using Mathcad Prime Express for maximum clarity and transparency.

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When a section is defined as a series of n nodes (x_i, y_i), HP calculator libraries use the following discrete integration formulas to calculate properties relative to the global origin:

Background of the method used for calculating section properties.

Section properties are determined by evaluating line integrals along the closed polygonal path defined by the section's vertices (nodes).

1. Cross-Sectional Area (A)

Also known as the Shoelace Formula:

$$A = \frac{1}{2}\sum_{i=1}^{n}(x_i y_{i+1} - x_{i+1} y_i)$$

2. Centroidal Coordinates ($\bar{x}, \bar{y}$)

Calculated using the first moments of area:

$$\bar{x} = \frac{1}{6A}\sum_{i=1}^{n}(x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i)$$

$$\bar{y} = \frac{1}{6A}\sum_{i=1}^{n}(y_i + y_{i+1})(x_i y_{i+1} - x_{i+1} y_i)$$

3. Moments of Inertia (I_x, I_y)

The second moments of area about the reference origin:

$$I_x = \frac{1}{12}\sum_{i=1}^{n}(y_i^2 + y_i y_{i+1} + y_{i+1}^2)(x_i y_{i+1} - x_{i+1} y_i)$$

$$I_y = \frac{1}{12}\sum_{i=1}^{n}(x_i^2 + x_i x_{i+1} + x_{i+1}^2)(x_i y_{i+1} - x_{i+1} y_i)$$

Why "Node Path" Integration?

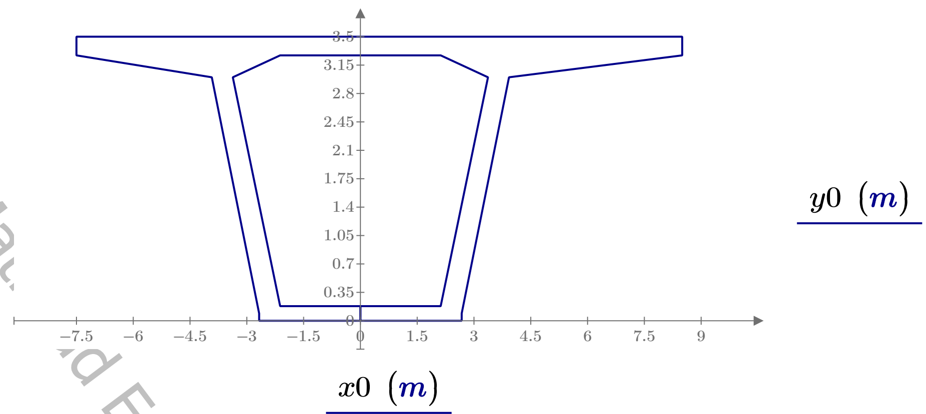
This method is superior for calculators and software for three reasons:

1. **Arbitrary Shapes:** It works for any non-self-intersecting polygon, whether convex or concave (like an L-angle or C-channel).
2. **Efficiency:** Instead of complex double integrals over the area, it only requires a single loop through the list of vertex coordinates.
3. **Accuracy:** It provides an exact analytical solution for polygonal shapes, unlike "pixel-counting" or mesh-based approximations.

Calculating section properties described by nodes

No	$x0$ (m)	$y0$ (m)
0	0	0
1	−2.675	0
2	−2.675	0.09
3	−3.925	3
4	−7.5	3.27
5	−7.5	3.5
6	8.5	3.5
7	8.5	3.27
8	3.925	3
9	2.675	0.09
10	2.675	0
11	0	0
12	0	0.18
13	2.12	0.18
14	3.37	3
15	2.12	3.27
16	−2.12	3.27
17	−3.37	3
18	−2.12	0.18
19	0	0.18
20	0	0

1. Descirbe an outer contour in clockwise direction, while voids in counter clockwise direction.
2. All contours shall start and end at 0,0



xo and yo are vectors for the 1st node of the segment.
Create vectors for the 2nd node:

$np := \text{rows}(x0) = 21$

$i := 0 .. np-2$

$x1_i := x0_{i+1}$ $\quad y1_i := y0_{i+1}$

$x1_{np-1} := x0_0$ $\quad y1_{np-1} := y0_0$

Note:
Multiplying two vectors in Mathcad calculates their dot product, resulting in a single scalar value equal to the sum of the products of their corresponding elements. $u \cdot v = \sum_{j=1}^{n} u_j \cdot v_j$

Dimensions across: $dX := \max(x0) - min(x0) = 16\ m$

$dY := \max(y0) - min(y0) = 3.5\ m$

Area of Cross-section: $A := 0.5 \cdot (y0 - y1) \cdot (x0 + x1) = 9.623\ m^2$

Peremeter: $P := \sum \sqrt{(y0-y1)^2 + (x0-x1)^2} = 54.059\ m$

Center of gravity from reference axes:

$$xc := \frac{1}{A} \cdot \left(\frac{1}{8} \cdot (y0 - y1) \cdot \left((x0 + x1)^2 + \frac{1}{3} \cdot (x1 - x0)^2 \right) \right) = 0.284\ m$$

$$yc := \frac{1}{A} \cdot \left(\frac{1}{8} \cdot (x1 - x0) \cdot \left((y0 + y1)^2 + \frac{1}{3} \cdot (y1 - y0)^2 \right) \right) = 2.406\ m$$

Section properties about axes parallel to the reference axes with origin at section centroid

Moment of inertia

$$Ix := \frac{1}{12} \cdot \overrightarrow{(x1 - x0) \cdot (y0 + y1)} \cdot \left(y1^2 + y0^2\right) - A \cdot yc^2 = 14.144 \; m^4$$

$$Iy := \frac{1}{12} \cdot \overrightarrow{(y0 - y1) \cdot (x0 + x1)} \cdot \left(x1^2 + x0^2\right) - A \cdot xc^2 = 150.123 \; m^4$$

Product moment of inertia

$$Ixy := \left(\overrightarrow{\left(\begin{array}{l} \frac{1}{8} \cdot (y1 - y0)^2 \cdot (x0 + x1) \cdot \left(x1^2 + x0^2\right) \\ + \frac{1}{3} \cdot (y1 - y0) \cdot (x1 \cdot y0 - x0 \cdot y1) \cdot \left(x1^2 + x1 \cdot x0 + x0^2\right) \\ + \frac{1}{4} \cdot (x1 \cdot y0 - x0 \cdot y1)^2 \cdot (x1 + x0) \end{array} \right) \cdot \left(\frac{(x0 \neq x1)}{(x1 - x0) + (x1 = x0) \cdot m} \right)} \right) - A \cdot xc \cdot yc = 2.503 \; m^4$$

Elastic section moduli

$$\begin{bmatrix} Sx.top \\ Sx.bot \end{bmatrix} := Ix \div \begin{bmatrix} \max(y0) - yc \\ \min(y0) - yc \end{bmatrix} = \begin{bmatrix} 12.931 \\ -5.878 \end{bmatrix} m^3$$

$$\begin{bmatrix} Sy.left \\ Sy.right \end{bmatrix} := Iy \div \begin{bmatrix} \min(x0) - xc \\ \max(x0) - xc \end{bmatrix} = \begin{bmatrix} -19.285 \\ 18.273 \end{bmatrix} m^3$$

Section properties about principal axes

Angle between principal and reference axes

$$\theta p := \text{if}\left(\text{round}\left(\frac{Ixy}{m^4}, 4\right) = 0, 0, \frac{1}{2} \cdot \text{atan}\left(-2 \cdot \frac{Ixy}{Ix - Iy}\right)\right) = 1.054 \; deg$$

Moment of inertia

$$Ixp := Ix \cdot \cos(\theta p)^2 + Iy \cdot \sin(\theta p)^2 - Ixy \cdot \sin(2 \cdot \theta p) = 14.098 \; m^4$$

$$Iyp := Ix + Iy - Ixp = 150.17 \; m^4$$

Polar moment of inertia

$$Ip := Ixp + Iyp = 164.268 \; m^4$$

Transforme node coordinates from reference to principal axes

$$xp_i := \left(x0_i - xc\right) \cdot \cos(\theta p) + \left(y0_i - yc\right) \cdot \sin(\theta p)$$

$$yp_i := -\left(x0_i - xc\right) \cdot \sin(\theta p) + \left(y0_i - yc\right) \cdot \cos(\theta p)$$

Section moduli

$$\begin{bmatrix} Sxp.top \\ Sxp.bot \end{bmatrix} := Ixp \div \begin{bmatrix} \max(yp) \\ \min(yp) \end{bmatrix} = \begin{bmatrix} 11.398 \\ -5.755 \end{bmatrix} m^3$$

$$\begin{bmatrix} Syp.left \\ Syp.right \end{bmatrix} := Iyp \div \begin{bmatrix} \min(xp) \\ \max(xp) \end{bmatrix} = \begin{bmatrix} -19.334 \\ 18.237 \end{bmatrix} m^3$$

Summary of Member Properties

$$x0 = \begin{bmatrix} 0 \\ -2.675 \\ -2.675 \\ -3.925 \\ -7.5 \\ -7.5 \\ 8.5 \\ 8.5 \\ 3.925 \\ 2.675 \\ 2.675 \\ 0 \\ 0 \\ 2.12 \\ \vdots \end{bmatrix} m \qquad y0 = \begin{bmatrix} 0 \\ 0 \\ 0.09 \\ 3 \\ 3.27 \\ 3.5 \\ 3.5 \\ 3.27 \\ 3 \\ 0.09 \\ 0 \\ 0 \\ 0.18 \\ \vdots \end{bmatrix} m$$

Area	$A = 9.623\ m^2$
Perimeter	$P = 54.059\ m$
Width	$dX = 16\ m$
Height	$dY = 3.5\ m$
Center of gravity	$yc = 2.406\ m$
	$xc = 0.284\ m$

Properties about axes parallel to original and passing through center of gravity

$Ix = 14.144\ m^4$
$Iy = 150.123\ m^4$
$Ixy = 2.503\ m^4$
$Ip = 164.268\ m^4$

$Sx.top = 12.931\ m^3$ $\quad Sx.bot = -5.878\ m^3$
$Sy.left = -19.285\ m^3$ $\quad Sy.right = 18.273\ m^3$

Properties about principal axes

$\theta p = 1.054\ deg$
$Ixp = 14.098\ m^4$
$Iyp = 150.17\ m^4$
$Ip = 164.268\ m^4$

$Sxp.top = 11.398\ m^3$ $\quad Sxp.bot = -5.755\ m^3$
$Syp.left = -19.334\ m^3$ $\quad Syp.right = 18.237\ m^3$

Enter company name	Project: PROJECT NAME	Engineer: Date: 6-Jan
SectProp 15-Day Demo	Subject: BOX-GIRDER SECTION	Checker: Date:

PROPERTIES OF SECTION DESCRIBED BY NODES COORDINATES

Units: **m**

Nodes Coordinates

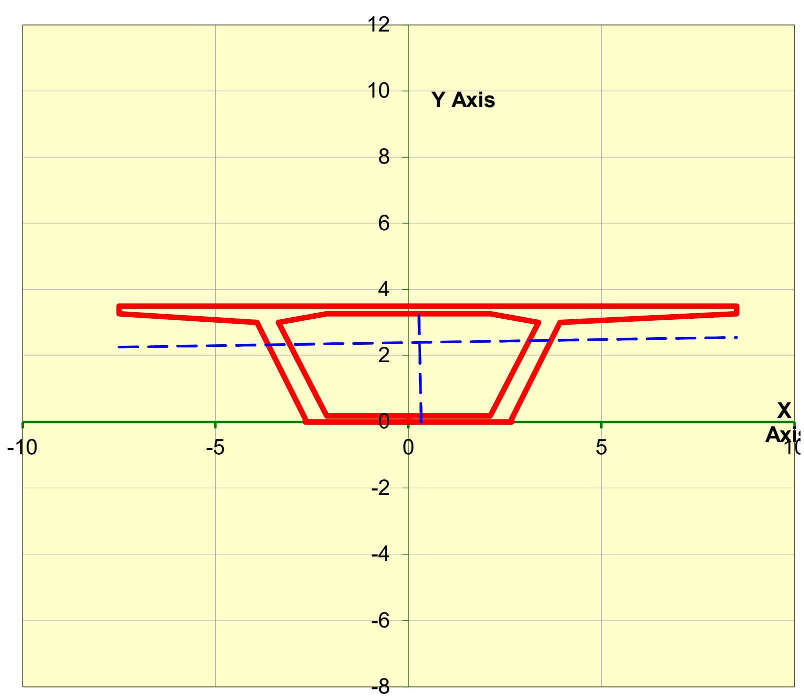
	x	y
1	0.000	0.000
2	-2.675	0.000
3	-2.675	0.090
4	-3.925	3.000
5	-7.500	3.270
6	-7.500	3.500
7	8.500	3.500
8	8.500	3.270
9	3.925	3.000
10	2.675	0.090
11	2.675	0.000
12	0.000	0.000
13	0.000	0.180
14	2.120	0.180
15	3.370	3.000
16	2.120	3.270
17	-2.120	3.270
18	-3.370	3.000
19	-2.120	0.180
20	0.000	0.180
21	0.000	0.000
22		
23		
24		
25		
26		
27		
28		
29		
30		
31		

Section properties about axes parallel to reference axes with origin at section centroid

A =	9.62315 m²	W =	16 m
P =	54.0592676 m	H =	3.5 m
x_c =	0.28437934 m	S_x^{top} =	12.93075 m³
y_c =	2.40614742 m	S_x^{bot} =	5.878416 m³
I_x =	14.1443359 m^4	S_y^{left} =	19.28522 m³
I_y =	150.123493 m^4	S_y^{right} =	18.27293 m³
I_{xy} =	2.5031962 m^4		

Section properties about principal axes

θ_p =	1.05426324 deg	S_{xp}^{top} =	11.39812 m³
I_{xp} =	14.098271 m^4	S_{xp}^{bot} =	5.75504 m³
I_{yp} =	150.169558 m^4	S_{yp}^{left} =	19.33389 m³
I_p =	164.267829 m^4	S_{yp}^{right} =	18.23695 m³



Calculating section properties described by layers

bt (in)	bb (in)	t (in)
49	49	3
30	12	3
12	6	3
6	6	28.375
6	12	3
12	38.5	4.5
38.5	38.5	4.125
38.5	36.5	1

Describe layers from top to bottom

t - thickness
bt - width at top
bb - width at bottom

$np := \text{rows}(t) = 8 \qquad i := 1..np-1$

$ht_0 := 0 \cdot in \qquad ht_i := ht_{i-1} + t_{i-1}$

Section properties

Height $H := \sum t = 50\ in$

Area $A := 0.5 \cdot t \cdot (bt + bb) = 744.188\ in^2$

C.G. from top $y_t := t \cdot \overrightarrow{\dfrac{(bt + 2 \cdot bb) \cdot t + 3 \cdot (bt + bb) \cdot ht}{6 \cdot A}} = 26.725\ in$

C.G. from bottom $y_b := H - y_t = 23.275\ in$

Moment of inertia about horizontal axis

$$I_x := t \cdot \left(\overrightarrow{bt \cdot \left(ht + \frac{t}{2} - y_t\right)^2 + \frac{(bb + 2 \cdot bt) \cdot t^2}{36} + \frac{bb - bt}{2} \cdot \left(ht + \frac{2}{3} \cdot t - y_t\right)^2} \right) = 268147.391\ in^4$$

Moment of inertia about vertical axis

$$I_y := \frac{t}{144} \cdot \left(\overrightarrow{12 \cdot bt^3 + (bb - bt) \cdot t^2 + 2 \cdot (bb - bt) \cdot (bb + 2 \cdot bt)^2} \right) = 64335.471\ in^4$$

Elastic section modulus, top $S_{xt} := \dfrac{I_x}{y_t} = 10033.47\ in^3$ bottom $S_{xb} := \dfrac{I_x}{y_b} = 11520.976\ in^3$