Design of RC member for combined action of axial force, bending, shear and torsion in accordance with AASHTO LRFD 10th Edition

Material properties

Strength: $\begin{bmatrix} f'_c \\ f_y \end{bmatrix} := \begin{bmatrix} 4 \\ 60 \end{bmatrix} \cdot ksi$

Max. concrete strain: $\varepsilon_c := -0.003$

Concrete unit weight: $w_c := 0.145 \cdot kcf$ (for stiffness)

Modulus of elasticity:

$$\begin{bmatrix} E_c \\ E_s \end{bmatrix} := \begin{bmatrix} 120000 \cdot \left(\frac{w_c}{kcf}\right)^2 \cdot \left(\frac{f'_c}{ksi}\right)^{0.33} \\ 29000 \end{bmatrix} \cdot ksi = \begin{bmatrix} 3987 \\ 29000 \end{bmatrix} ksi \qquad n := \frac{E_s}{E_c} = 7.27$$

Equivalent compression zone coefficient: $\beta_1 := \max\left(0.65, 0.85 - \left(\frac{f'_c}{ksi} > 4\right) \cdot 0.05 \cdot \left(\frac{f'_c}{ksi} - 4\right)\right) = 0.85$

Stress block factor: $\alpha_1 := \max\left(0.75, 0.85 - \left(\frac{f'_c}{ksi} > 10\right) \cdot 0.02 \cdot \left(\frac{f'_c}{ksi} - 10\right)\right) = 0.85$

Resistance factors for flexure and shear $\begin{bmatrix} \phi_t \\ \phi_c \\ \phi_v \end{bmatrix} := \begin{bmatrix} 0.9 \\ 0.75 \\ 0.9 \end{bmatrix}$

Compression () and tension () controlled strain limits in reinforcement per Section 5.6.2.1

$$\varepsilon_{cl} := 0.002 + 0.002 \cdot min\left(1, \max\left(0, \frac{\frac{f_y}{ksi} - 60}{40}\right)\right) = 0.002$$

$$\varepsilon_{tl} := 0.005 + 0.003 \cdot min\left(1, \max\left(0, \frac{\frac{f_y}{ksi} - 75}{25}\right)\right) = 0.005$$

$SectionType :=$ "CIRCULAR" $= 1$

Rectangular section parameters $\text{if}(SectionType = 2, \text{“Applicable”}, \text{“N/A”}) = \text{“N/A”}$

Width $B := 36 \cdot in$
Height $H := 48 \cdot in$
Tie bar diameter $d_{tb} := 0.75 \cdot in$
Side clear to tie bar $clr_{tb} := 1.5 \cdot in$
Reinforcing layers
Distance from the bottom concrete face to c.g. of the layer - y_{s2}
Cross-sectional area of each bar in the layer - A_{sb}
Number of bars per layer - n_{b2}

y_{s2} (in)	Bar	n_{b2}
3	#6	5
13	#6	2
23	#6	2
33	#6	2
44	#9	7

$A_{sb} := \overrightarrow{Asb(Bar)} = ?\ in^2$ $d_{bo} := \overrightarrow{dbo(Bar)} = ?\ in$
Reinforcing bar maximum overall diameter
$d_{b.max} := \max(d_{bo}) = ?\ in$

Distance between outer bars: $b_b := B - 2 \cdot (clr_{tb} + d_{tb} + 0.5 \cdot d_{b.max}) = 30.26\ in$
Number of bars at the bottom: $n_{b.bot} := \text{lookup}(\max(y_{s2}), y_{s2}, n_{b2})_{\text{ORIGIN}} = 7$
Number of bars at the top: $n_{b.top} := \text{lookup}(min(y_{s2}), y_{s2}, n_{b2})_{\text{ORIGIN}} = 5$
Spacing of outer bars: $\begin{bmatrix} s_{bot} \\ s_{top} \end{bmatrix} := b_b \div \begin{bmatrix} n_{b.bot} - 1 \\ n_{b.top} - 1 \end{bmatrix} = \begin{bmatrix} 5.043 \\ 7.565 \end{bmatrix} in$

Circular section parameters $\text{if}(SectionType = 1, \text{“Applicable”}, \text{“N/A”}) = \text{“Applicable”}$

Section diameter: $D := 42 \cdot in$ radius $R := 0.5 \cdot D$
Cover to hoop bar: $clr := 2 \cdot in$
Number of longitudinal bars: $n_b := 16$

$\begin{bmatrix} Long_bar \\ Hoop_bar \end{bmatrix} := \begin{bmatrix} \#10 \\ \#5 \end{bmatrix}$

Cross-section area of bars: $\begin{bmatrix} A_{bl} \\ A_{bh} \end{bmatrix} := \overrightarrow{Asb\left(\begin{bmatrix} Long_bar \\ Hoop_bar \end{bmatrix}\right)} = \begin{bmatrix} 1.27 \\ 0.31 \end{bmatrix} in^2$

Overall diameter of bars: $\begin{bmatrix} d_{bl} \\ d_{bh} \end{bmatrix} := \overrightarrow{dbo\left(\begin{bmatrix} Long_bar \\ Hoop_bar \end{bmatrix}\right)} = \begin{bmatrix} 1.4 \\ 0.7 \end{bmatrix} in$

Bar spacing: $s_{circ} := \dfrac{\pi \cdot (D - 2 \cdot (clr + d_{bh} + 0.5 \cdot d_{bl}))}{n_b} = 6.91\ in$

Section properties

$A_g := \text{if}\left(SectionType = 1, 0.25 \cdot \pi \cdot D^2, B \cdot H\right) = 1385.4\ in^2$

$I_g := \text{if}\left(SectionType = 1, \frac{\pi \cdot D^4}{64}, \frac{B \cdot H^3}{12}\right) = 152745\ in^4$

$h := \text{if}\left(SectionType = 1, D, H\right) = 42\ in$

Total area of reinforcement: $\sum A_s = 20.32\ in^2$

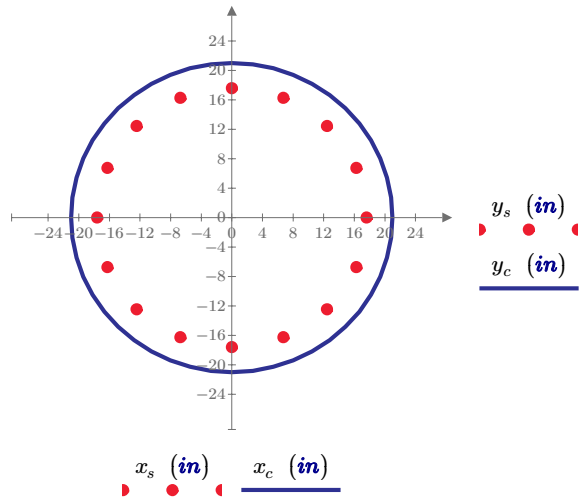
$n_b = 16 \quad \rho_s := \frac{\sum A_s}{A_g} = 1.47\%$

Transformed cross-section properties

$A_t := A_g + \left(\sum A_s\right) \cdot \left(\frac{E_s}{E_c} - 1\right) = 1513\ in^2$

$y_t := \frac{\sum_{i=0}^{n_b - 1} A_{s_i} \cdot y_{s_i}}{A_t} = 0\ in$

$I_t := I_g + \left(\frac{E_s}{E_c} - 1\right) \cdot \sum_{i=0}^{n_b - 1} A_{s_i} \cdot y_{s_i}^{\ 2} = 172492\ in^4$



Functions for strain/stress compatibility analysis

$$fs(c, ds) := \left\| \begin{array}{l} \text{"Stress in rebar"} \\ es \leftarrow \varepsilon_c \cdot \left(1 - \frac{ds}{c}\right) \\ fs \leftarrow \text{sign}(es) \cdot \text{if } |es \cdot E_s| \geq f_y \;\; + \text{if } ds < c \cdot \beta_1 \\ \qquad \left\| f_y \right. \qquad \left\| 0.85 \cdot f'_c \right. \\ \qquad \text{else} \qquad \text{else} \\ \qquad \left\| |es \cdot E_s| \right. \qquad \left\| 0 \cdot ksi \right. \end{array} \right.$$

$$ds(m) := \left\| \begin{array}{l} \text{"returns distance to reinforcing "} \\ \text{"bar from compressive edge"} \\ \text{"m > 0 for positive moment"} \\ \text{"m < 0 for negative"} \\ \text{return } 0.5 \cdot h - \text{sign}(m) \cdot y_s \end{array} \right.$$

$$\phi f(\varepsilon_t) := \text{if } \varepsilon_t \geq \varepsilon_{tl} \\ \left\| \phi_t \right. \\ \text{else if } \varepsilon_t \leq \varepsilon_{cl} \\ \left\| \phi_c \right. \\ \text{else} \\ \left\| \phi_c + (\phi_t - \phi_c) \cdot \frac{\varepsilon_t - \varepsilon_{cl}}{\varepsilon_{tl} - \varepsilon_{cl}} \right.$$

Concrete stress $fc(x) := \alpha_1 \cdot f'_c$
(assumed constant $\alpha_1 \cdot f'_c = 3.4\ ksi$

Section width
$b(x) := \text{if}\left(SectionType = 2, B, 2 \cdot \sqrt{D \cdot x - x^2}\right)$

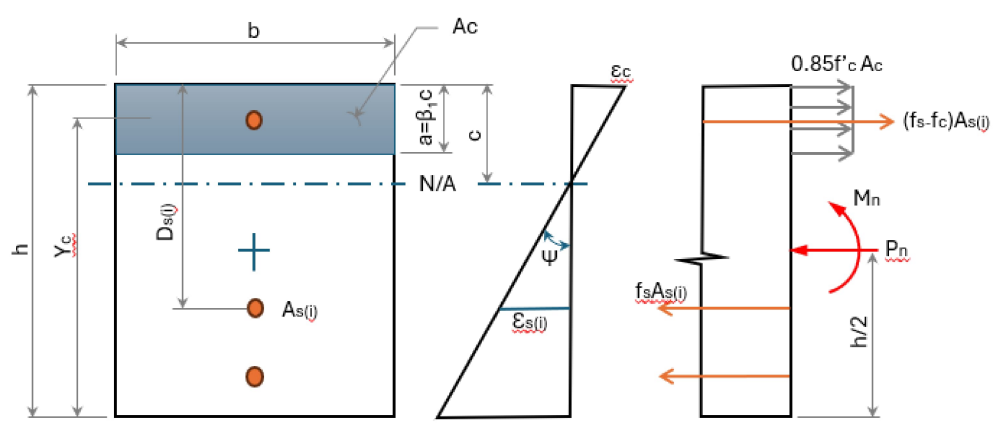
where x is the distance from the edge in compression

Resultant of concrete compressive force:

Axial $Pc(c) := \int_{0 \cdot ft}^{min(c \cdot \beta_1, h)} fc(x) \cdot b(x)\, dx$

Moment $Mc(c) := \int_{0 \cdot ft}^{min(c \cdot \beta_1, h)} fc(x) \cdot b(x) \cdot \left(\frac{h}{2} - x\right) dx$

$$PM2(c, d_s) := \left\| \begin{array}{l} \text{"returns nominal axial and moment"} \\ \text{" resistance corresponding to"} \\ \text{" location of Neutral Axis – c"} \\ \left\| \begin{array}{l} \text{for } i \in 0 .. n_b - 1 \\ \quad \left\| f_{s_i} \leftarrow fs(c, d_{s_i}) \right. \\ Pn \leftarrow Pc(c) - \sum_{i=0}^{n_b - 1} \left(A_{s_i} \cdot f_{s_i}\right) \\ Mn \leftarrow Mc(c) + \sum_{i=0}^{n_b - 1} A_{s_i} \cdot f_{s_i} \cdot \left(d_{s_i} - \frac{h}{2}\right) \end{array} \right. \\ \text{return } \begin{bmatrix} Pn \\ Mn \end{bmatrix} \end{array} \right.$$



Number of points on interaction diagram $Np := 30$ $j := 0..Np$, bar counter $i := 0..n_b - 1$

Gradually increase distance c from compression edge to neutral axis

$c_j := \text{if}\left(j = 0, 0.01 \cdot in, \frac{2 \cdot h \cdot j}{\beta_1 \cdot Np}\right)$ Curvature $\psi_j := \frac{\varepsilon_c}{c_j}$

Nominal resistance

Maximum tension $P_{nt} := -f_y \cdot \sum A_s = -1219.2\ kip$

Maximum compression

$P_{nc} := (0.8 + 0.05 \cdot (SectionType = 1)) \cdot \left(0.85 \cdot f'_c \cdot \left(A_g - \sum A_s\right) + f_y \cdot \sum A_s\right) = 4981.5\ kip$ (5.6.4.4-2, 3)

Positive moment **Negative moment**

Depth to reinforcement

$d_{s.pos} := ds(1)$ $d_{s.neg} := ds(-1)$

Axial force (+ for compression)

$P_{n.pos_j} := PM2\left(c_j, d_{s.pos}\right)_0$ $P_{n.neg_j} := PM2\left(c_j, d_{s.neg}\right)_0$

Nominal Bending Moment Resistance

$M_{n.pos_j} := PM2\left(c_j, d_{s.pos}\right)_1$ $M_{n.neg_j} := -PM2\left(c_j, d_{s.neg}\right)_1$

Balance point

$M_{nb.pos} := \max\left(M_{n.pos}\right) = 2637.55\ kip \cdot ft$ $M_{nb.neg} := min\left(M_{n.neg}\right) = -2637.55\ kip \cdot ft$

$P_{nb.pos} := P_{n.pos_{\text{match}\left(M_{nb.pos}, M_{n.pos}\right)_0}} = 2241.9\ kip$ $P_{nb.neg} := P_{n.neg_{\text{match}\left(M_{nb.neg}, M_{n.neg}\right)_0}} = 2241.9\ kip$

Full compression (when c = h)

$M_{nc.pos} := \text{linterp}\left(c, M_{n.pos}, h\right) = 1041.4\ kip \cdot ft$ $M_{nc.neg} := \text{linterp}\left(c, M_{n.neg}, h\right) = -1041.4\ kip \cdot ft$

$P_{ncf.pos} := \text{linterp}\left(c, P_{n.pos}, h\right) = 4997.1\ kip$ $P_{ncf.neg} := \text{linterp}\left(c, P_{n.neg}, h\right) = 4997.1\ kip$

Resistance factor per Section 5.6.2.1. Maximum net tensile strain in reinforcing

$\varepsilon_{t.pos} := \varepsilon_c \cdot \left(1 - \frac{\max\left(d_{s.pos}\right)}{c}\right)$ $\varepsilon_{t.neg} := \varepsilon_c \cdot \left(1 - \frac{\max\left(d_{s.neg}\right)}{c}\right)$

Resistance factor is interpolated for ε_t between $\varepsilon_{tl} = 0.005$ (0.9) and $\varepsilon_{cl} = 0.002$ (0.75)

$\phi_{pos_j} := \phi f\left(\varepsilon_{t.pos_j}\right)$ $\phi_{neg_j} := \phi f\left(\varepsilon_{t.neg_j}\right)$

Factored resistance

$P_{r.pos_j} := \phi_{pos_j} \cdot min\left(P_{n.pos_j}, j \cdot 10^{-6} \cdot kip + P_{nc}\right)$ $P_{r.neg_j} := \phi_{neg_j} \cdot min\left(P_{n.neg_j}, j \cdot 10^{-6} \cdot kip + P_{nc}\right)$

$M_{r.pos} := \overrightarrow{\phi_{pos} \cdot M_{n.pos}}$ $M_{r.neg} := \overrightarrow{\phi_{neg} \cdot M_{n.neg}}$

Combine results for positive and negative bending in a single array:

$P_n := \text{stack}\left(\text{reverse}\left(P_{n.neg}\right), P_{n.pos}\right)$ $M_n := \text{stack}\left(\text{reverse}\left(M_{n.neg}\right), M_{n.pos}\right)$

$P_r := \text{stack}\left(\text{reverse}\left(P_{r.neg}\right), P_{r.pos}\right)$ $M_r := \text{stack}\left(\text{reverse}\left(M_{r.neg}\right), M_{r.pos}\right)$

$\phi := \text{stack}\left(\text{reverse}\left(\phi_{neg}\right), \phi_{pos}\right)$

Calculate Demand/Capacity Ratios

Acting Loads

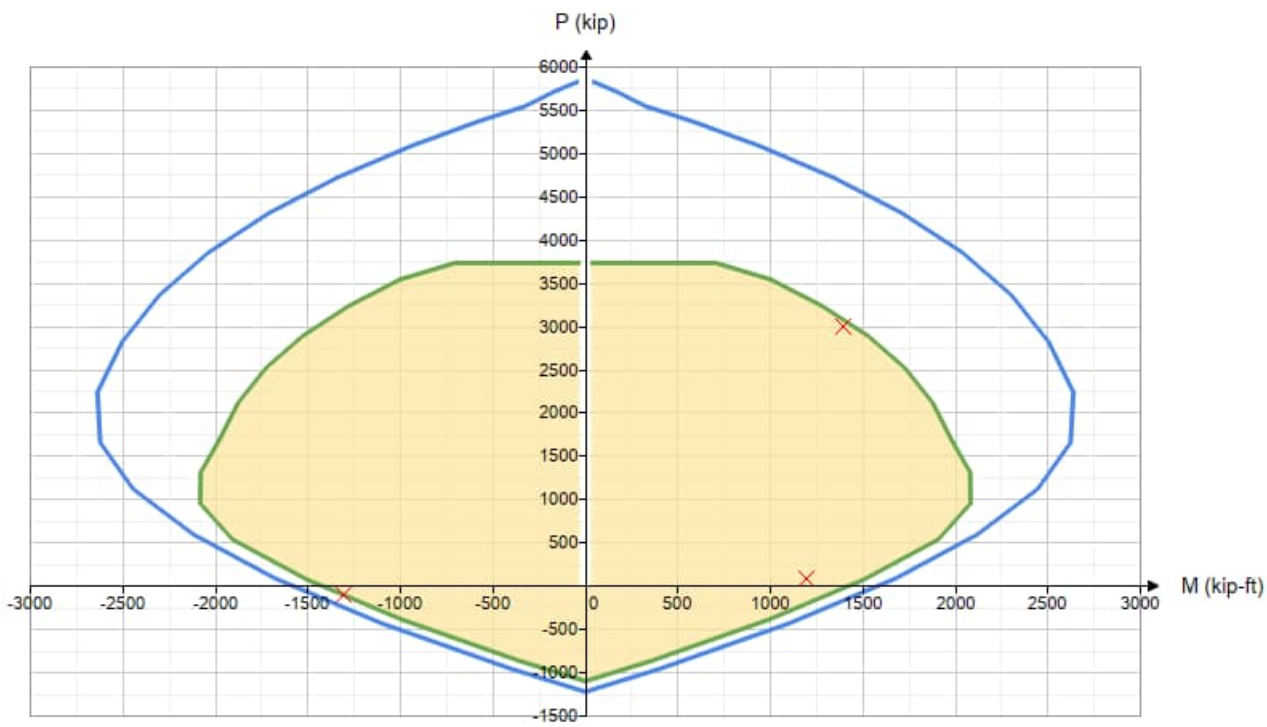
$Load_Case$	P_u (kip)	M_u $(kip \cdot ft)$
"Str1"	3000	1400
"P max"	70	1200
"P min"	−100	−1300

$$Mr(Pu, Mu) := \begin{array}{l} \text{if } Mu \geq 0 \\ \quad \text{"Interpolate Mr for given Pu"} \\ \quad \text{linterp}(P_{r.pos}, M_{r.pos}, Pu) \\ \text{else} \\ \quad \text{linterp}(P_{r.neg}, M_{r.neg}, Pu) \end{array}$$

$$M_{r1} := \overrightarrow{Mr(P_u, M_u)} = \begin{bmatrix} 1451 \\ 1496 \\ -1308 \end{bmatrix} kip \cdot ft$$

$$DCR_{PM} := \frac{M_u}{M_{r1}} = \begin{bmatrix} 0.965 \\ 0.802 \\ 0.994 \end{bmatrix} \quad \overrightarrow{Check(DCR_{PM} \leq 1)} = \begin{bmatrix} \text{"Good"} \\ \text{"Good"} \\ \text{"Good"} \end{bmatrix}$$

P/M Interaction Diagram



Service Load Analysis Functions

To find steel stresses under service load requires to solve two equilibrium equations with two unknowns:
c - location of the neutral axis from compressive face and
f_c - maximum value of triangular concrete compressive stress

Write functions to be later used in the solver block:

Stress in reinforcing bars: $f_s(c, f_c, d_s) := \overrightarrow{f_c \cdot \left(1 - \frac{d_s}{c}\right) \cdot \left(\frac{E_s}{E_c} - (d_s \le c)\right)}$

Describe the concrete area in compression and stress diagram as a function of
x - distance from compression fiber

Section width $b(x) := \text{if}\left(SectionType = 2, B, 2 \cdot \sqrt{D \cdot x - x^2}\right)$

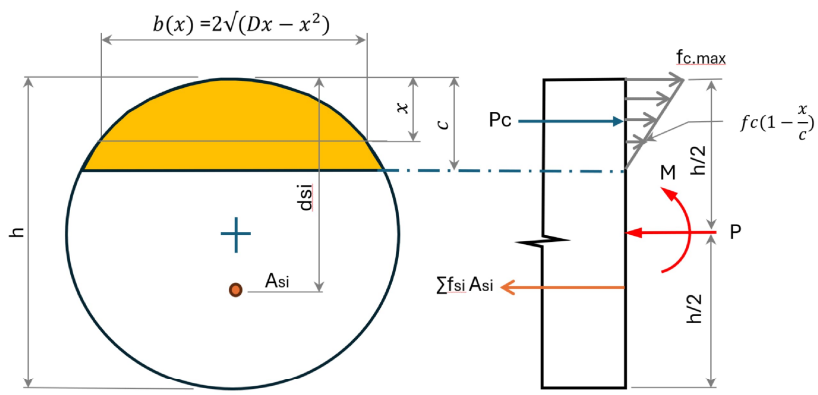
Triangular Concrete stress diagram $fc(x, c, f_c) := f_c \cdot \left(1 - \frac{x}{c}\right)$

Then compressive force in the concrete written as a function of c and fc:

$$P_c(c, f_c) := \int_{0 \cdot ft}^{c} b(x) \cdot fc(x, c, f_c)\, dx$$

Moment about the section's centroid:

$$M_c(c, f_c) := \int_{0 \cdot ft}^{c} b(x) \cdot fc(x, c, f_c) \cdot \left(\frac{h}{2} - x\right) dx$$



Write axial force and bending moment equilibrium equations in the solver block. Write a function for solving the system of equations for acting P and M.

Write a function solving equilibrium equations $\sum P = 0$ and $\sum M = 0$ for c_{NA} and $f_{c.max}$

Guess Values

$c_{NA} := 0.33 \cdot h$ $f_{c.max} := -2 \cdot ksi$ (Initial values)

Constraints

$P_c(c_{NA}, f_{c.max}) + A_s \cdot f_s(c_{NA}, f_{c.max}, d_s) = -P$

$M_c(c_{NA}, f_{c.max}) + \overrightarrow{A_s \cdot f_s(c_{NA}, f_{c.max}, d_s)} \cdot \overrightarrow{(0.5 \cdot h - d_s)} = -|M|$

Solver

$cf(P, M, d_s) := \text{Find}(c_{NA}, f_{c.max})$

Crack Control per AASHTO Section 5.6.7

Service Loads:

$$P := \begin{bmatrix} 200 \\ 100 \\ 0 \end{bmatrix} \cdot kip \qquad M := \begin{bmatrix} 900 \\ -600 \\ 500 \end{bmatrix} \cdot kip \cdot ft$$

Find tensile stresses in outer reinforcing bars (f_{ss}):

Solve for each load: $i := \mathrm{ORIGIN} .. \mathrm{last}(P)$

For M < 0, change ds to h-ds

$$c_{NA_i} := cf\left(P_i, M_i, ds\left(\mathrm{sign}\left(M_i\right)\right)\right)_{\mathrm{ORIGIN}} = \begin{bmatrix} 13.57 \\ 13 \\ 11.46 \end{bmatrix} in$$

$$f_{c.max_i} := cf\left(P_i, M_i, ds\left(\mathrm{sign}\left(M_i\right)\right)\right)_{\mathrm{ORIGIN}+1} = \begin{bmatrix} -2.643 \\ -1.777 \\ -1.505 \end{bmatrix} ksi$$

$$f_{ss_i} := f_{c.max_i} \cdot \left(1 - \frac{\max\left(ds\left(\mathrm{sign}\left(M_i\right)\right)\right)}{c_{NA_i}}\right) \cdot \left(\frac{E_s}{E_c} - \left(\max\left(ds\left(\mathrm{sign}\left(M_i\right)\right)\right) \le c_{NA_i}\right)\right) = \begin{bmatrix} 35.47 \\ 25.46 \\ 25.91 \end{bmatrix} ksi$$

Exposure factor: $\gamma_e :=$ Exposure Class: 2 (more severe) $= 0.75$

Distance from tensile reinf. to nearest concrete fiber: $d_{c_i} := h - \max\left(ds\left(M_i\right)\right) = \begin{bmatrix} 3.4 \\ 3.4 \\ 3.4 \end{bmatrix} in$

Spacing of tensile reinforcement: $s_i := \begin{bmatrix} SectionType = 1 \\ SectionType = 2 \wedge M_i \ge 0 \\ SectionType = 2 \wedge M_i < 0 \end{bmatrix} \cdot \begin{bmatrix} s_{circ} \\ s_{bot} \\ s_{top} \end{bmatrix} = \begin{bmatrix} 6.912 \\ 6.912 \\ 6.912 \end{bmatrix} in$

Crack coefficient: $\beta_s := 1 + \dfrac{d_c}{0.7\ (h - d_c)} = \begin{bmatrix} 1.126 \\ 1.126 \\ 1.126 \end{bmatrix}$ (5.6.7-2)

$$s_{max} := \overrightarrow{\frac{700 \cdot \frac{kip}{in} \cdot \gamma_e}{\beta_s \cdot f_{ss}}} - 2 \cdot d_c = \begin{bmatrix} 6.35 \\ 11.52 \\ 11.2 \end{bmatrix} in \qquad (5.6.7\text{-}1)$$

$$\overrightarrow{Check\left(s \le s_{max} \wedge f_{ss} \le 0.6 \cdot f_y\right)} = \begin{bmatrix} \text{“N.G.”} \\ \text{“Good”} \\ \text{“Good”} \end{bmatrix}$$

Corresponding crack width: $\Delta_{cr} := \dfrac{\overrightarrow{\left(s + 2 \cdot d_c\right) \cdot \beta_s \cdot f_{ss} \cdot 0.017 \cdot in}}{700 \cdot \frac{kip}{in}} = \begin{bmatrix} 0.34 \\ 0.24 \\ 0.25 \end{bmatrix} mm$

Effective moment of inertia per service loads (per AASHTO Section 5.6.3.5.2)

For earlier defined loads: $P=\begin{bmatrix}200\\100\\0\end{bmatrix}\ kip\quad M=\begin{bmatrix}900\\-600\\500\end{bmatrix}\ kip\cdot ft$

Cracked section properties for rectangular section under service load:

$$A_{cr_i}:=B\cdot c_{NA_i}+A_s\cdot\overrightarrow{\left(\frac{E_s}{E_c}-\left(d_s\le c_{NA_i}\right)\right)}=\begin{bmatrix}629.926\\609.383\\554.169\end{bmatrix}\ in^2$$

$$y_{cr_i}:=\frac{0.5\cdot B\cdot c_{NA_i}{}^2+\overrightarrow{A_s\cdot d_s}\cdot\left(\frac{E_s}{E_c}-\left(d_s\le c_{NA_i}\right)\right)}{A_{cr_i}}=\begin{bmatrix}10.128\\10.022\\9.801\end{bmatrix}\ in$$

$$I_{cr_i}:=\frac{B\cdot c_{NA_i}{}^3}{12}+B\cdot c_{NA_i}\cdot\left(y_{cr_i}-0.5\cdot c_{NA_i}\right)^2+\overrightarrow{A_s\cdot\left(y_{cr_i}-d_s\right)^2}\cdot\left(\frac{E_s}{E_c}-\left(d_s\le c_{NA_i}\right)\right)=\begin{bmatrix}53184\\52972\\52665\end{bmatrix}\ in^4$$

$$\frac{I_{cr}}{I_g}=\begin{bmatrix}0.348\\0.347\\0.345\end{bmatrix}$$

Concrete modulus of rupture per 5.4.2.6 $\quad f_r:=0.24\cdot\sqrt{f'_c\cdot ksi}=0.48\ ksi$

Cracking moment $\quad M_{cr}:=\left(f_r+\frac{P}{A_g}\right)\cdot\frac{I_g}{0.5\cdot h}=\begin{bmatrix}378.44\\334.69\\290.94\end{bmatrix}\ kip\cdot ft$ (5.6.3.5.2-1 modified)

Effective moment of inertia:

$$I_{e_i}:=min\left(\left(\frac{M_{cr_i}}{\left|M_i\right|}\right)^3\cdot I_g+\left(1-\left(\frac{M_{cr_i}}{\left|M_i\right|}\right)^3\right)\cdot I_{cr_i},I_g\right)=\begin{bmatrix}60586\\70290\\72383\end{bmatrix}\ in^4\quad(5.6.3.5.2\text{-}1)$$

$$\frac{I_e}{I_g}=\begin{bmatrix}0.397\\0.46\\0.474\end{bmatrix}$$

Design for Shear and Torsion

Loads

$P_u := 300 \cdot kip$

$V_u := 300 \cdot kip$

$M_u := 1000 \cdot kip \cdot ft$

$T_u := 500 \cdot kip \cdot ft$

Resistance factors

$\phi_f := 0.9$

$\phi_v := 0.9$

Transverse reinforcing

Yield strength, $f_{yh} := 60 \cdot ksi$

Spacing of transverse reinforcing, $s := 6 \cdot in$

$Hoop_bar = 5$

$A_{bh} = 0.31\ in^2$

Number of hoops in the bundle, $n_h := 1$

For rectangular concrete cross-section:

Number of links in the direction of shear force:

$n_v := 2$

$Link_bar :=$ #5

$A_{vb} := Asb(Link_bar) = 0.31\ in^2$

Torsional properties

$$\begin{bmatrix} p_c \\ A_{cp} \\ p_h \\ A_{oh} \\ A_o \end{bmatrix} := \begin{array}{l} \text{if } SectionType = 1 \\ \quad p_c \leftarrow \pi \cdot D \\ \quad A_{cp} \leftarrow 0.25 \cdot \pi \cdot D^2 \\ \quad c_b \leftarrow clr + 0.5 \cdot d_{bh} \\ \quad p_h \leftarrow \pi \cdot (D - 2 \cdot c_b) \\ \quad A_{oh} \leftarrow 0.25 \cdot \pi \cdot (D - 2 \cdot c_b)^2 \\ \quad b_e \leftarrow \dfrac{A_{cp}}{p_c} \\ \quad A_o \leftarrow 0.25 \cdot \pi \cdot (D - b_e)^2 \\ \text{else} \\ \quad p_c \leftarrow 2 \cdot (B + H) \\ \quad A_{cp} \leftarrow B \cdot H \\ \quad c_b \leftarrow clr_{tb} + d_{tb} \\ \quad p_h \leftarrow p_c - 8 \cdot c_b \\ \quad A_{oh} \leftarrow (B - 2 \cdot c_b) \cdot (H - 2 \cdot c_b) \\ \quad b_e \leftarrow \dfrac{A_{cp}}{p_c} \\ \quad A_o \leftarrow (H - b_e) \cdot (B - b_e) \\ \text{return stack}(p_c, A_{cp}, p_h, A_{oh}, A_o) \end{array}$$

$$\begin{bmatrix} p_c \\ p_h \end{bmatrix} = \begin{bmatrix} 131.95 \\ 117.18 \end{bmatrix} in$$

$$\begin{bmatrix} A_{cp} \\ A_{oh} \\ A_o \end{bmatrix} = \begin{bmatrix} 1385.4 \\ 1092.7 \\ 779.3 \end{bmatrix} in^2$$

Area of torsional reinforcing $A_t := A_{bh} \cdot n_h = 0.31\ in^2$

Area of shear reinforcing

$A_v := \text{if}(SectionType = 1, 2 \cdot n_h \cdot A_{bh}, 2\ A_{bh} + n_v \cdot A_{vb}) = 0.62\ in^2$

Area of longitudinal reinforcing on flexural tension side $A_{st} := A_s \cdot (d_s \geq 0.5 \cdot h) = 11.43\ in^2$

Area of concrete on flexural tension side $A_{ct} := 0.5 \cdot A_g = 692.721\ in^2$

Effective depth for flexure $d_e := \dfrac{\overrightarrow{A_s \cdot (d_s \geq 0.5 \cdot h)} \cdot d_s}{A_{st}} = 30.831\ in$

Location of neutral axis $cna := \text{linterp}(\text{if}(M_u \geq 0, P_{r.pos}, P_{r.neg}), c, P_u) = 11.453\ in$

Effective depth for shear $d_v := \max\left(\begin{bmatrix} d_e - 0.5 \cdot cna \cdot \beta_1 \\ 0.9 \cdot d_e \\ 0.72 \cdot h \end{bmatrix}\right) = 30.24\ in$

Effective width for shear $b_v := \text{if}(SectionType = 1, D, B) = 42\ in$

Minimum bending moment to consider $M_u := \max(|M_u|, V_u \cdot d_v) = 1000\ kip \cdot ft$

Consideration of torsion

$$K := min\left(\sqrt{1+\frac{P_u}{A_g \cdot 0.126 \cdot \sqrt{f'_c \cdot ksi}}}, 2\right) = 1.364 \qquad (5.7.2.1\text{-}6)$$

$$T_{cr} := 0.126 \cdot K \cdot \sqrt{f'_c \cdot ksi} \cdot \frac{A_{cp}^{\ 2}}{p_c} = 416.6\ kip \cdot ft \qquad (5.7.2.1\text{-}4)$$

$$T_u = 500\ kip \cdot ft$$

$$Consider_torsion? := \left(T_u > 0.25 \cdot \phi_v \cdot T_{cr}\right) = 1 \qquad (5.7.2.1\text{-}3)$$

Calculate β and θ

$$V_{eff} := \sqrt{V_u^{\ 2} + Consider_torsion? \cdot \left(\frac{0.9 \cdot p_h \cdot T_u}{2 \cdot A_o}\right)^2} = 504.802\ kip \qquad (5.7.3.4.2\text{-}5)$$

Concrete shear stress $v_u := \dfrac{V_{eff}}{\phi_v \cdot b_v \cdot d_v} = 0.442\ ksi$ $\qquad \dfrac{v_u}{f'_c} = 0.11$

Tensile strain at centroid of A_{st}

$$\varepsilon_s := \frac{\dfrac{M_u}{d_v} - 0.5 \cdot P_u + V_{eff}}{E_s \cdot A_{st} + E_c \cdot A_{ct} \cdot \left(\dfrac{M_u}{d_v} - 0.5 \cdot P_u + V_{eff} < 0\right)} = 0.002 \qquad (5.7.3.4.2\text{-}4)$$

$$\varepsilon_s := \max\left(\varepsilon_s, -0.0004\right) = 0.002$$

$$\beta := \frac{4.8}{1 + 750 \cdot \varepsilon_s} = 1.777 \qquad (5.7.3.4.2\text{-}1)$$

$$\theta := \left(29 + 3500 \cdot \varepsilon_s\right) \cdot deg = 36.936\ deg \qquad (5.7.3.4.2\text{-}3)$$

Concrete shear resistance $V_c := 0.0316 \cdot \beta \cdot \sqrt{f'_c \cdot ksi} \cdot b_v \cdot d_v = 142.7\ kip$

Minimum shear reinforcement requirements

$$Shear_reinf_required := \left(V_u > 0.5 \cdot \phi_v \cdot V_c\right) = 1$$

$$A_{v.min} := 0.0316 \cdot \sqrt{f'_c \cdot ksi} \cdot \frac{b_v \cdot s}{f_y} \cdot Shear_reinf_required = 0.265\ in^2 \qquad 5.7.2.5\text{-}1$$

$A_v = 0.62\ in^2$ $\qquad Check\left(A_v \geq A_{v.min}\right) = \text{“Good”}$

Check maximum allowable spacing of shear reinforcement

$$s_{max} := \text{if}\left(v_u < 0.125 \cdot f'_c, min\left(0.8 \cdot d_v, 24 \cdot in\right), min\left(0.4 \cdot d_v, 12 \cdot in\right)\right) = 24\ in$$

$s = 6\ in$ $\qquad Check\left(s \leq s_{max}\right) = \text{“Good”}$

Torsion resistance

$$T_r := \phi_v \cdot \frac{2 \cdot A_o \cdot A_t \cdot f_{yh} \cdot \cot(\theta)}{s} = 482\ kip \cdot ft$$ (5.7.3.6.2-1)

Demand/capacity ratio $DCR_{torsion} := \frac{T_u}{T_r} = 1.037$ $Check\left(\frac{T_u}{T_r} \le 1\right) = \text{"N.G."}$

Shear resistance

Calculate hoop reinforcement required for torsion

$$A_{t.req'd} := Consider_torsion? \cdot \frac{T_u}{T_r} \cdot A_t = 0.322\ in^2$$

Shear reinforcement after reduction for torsion

$$A_v := A_v - A_t \cdot \text{if}\left(SectionType = 1, \frac{\pi}{4}, 1\right) = 0.377\ in^2$$

Steel shear resistance $V_s := \frac{A_v \cdot f_{yh} \cdot d_v \cdot \cot(\theta)}{s} = 151.4\ kip$ (5.7.3.3-4)

Maximum shear resistance $V_{n.max} := 0.25 \cdot f'_c \cdot b_v \cdot d_v = 1270.1\ kip$ (5.7.3.3-2)

Factored resistance $V_r := \phi_v \cdot min(V_c + V_s, V_{n.max}) = 264.7\ kip$ (5.7.3.3-1, 2)

Demand/capacity ratio $DCR_{shear} := \frac{V_u}{V_r} = 1.133$ $Check\left(\frac{V_u}{V_r} \le 1\right) = \text{"N.G."}$

Longitudinal reinforcement requirements - 5.7.3.5 and 5.7.3.6.3

Areas of longitudinal reinforcement required for:

a) Flexure and axial force (1st half of (5.7.3.5-1) $A_{sf} := \frac{\left(\frac{M_u}{\phi_f \cdot d_v} + \frac{0.5 \cdot -P_u}{\phi_f}\right)}{f_y} = 4.571\ in^2$

b) Shear (2nd half of (5.7.3.5-1)) $A_{sv} := \frac{\left(\frac{V_u}{\phi_v} - 0.5 \cdot V_s\right) \cdot \cot(\theta)}{f_y} = 5.711\ in^2$

c) Torsion (distributed around perimeter, based on (5.7.3.6.3-1))

$$A_l := \left(\frac{0.45 \cdot p_h \cdot T_u}{2 \cdot A_o \cdot \phi_v}\right) \cdot \frac{\cot(\theta)}{f_y} = 5\ in^2$$

Total longitudinal reinforcement req'd on the flexural tension side of the member (5.7.3.6.3-1)

$$A_{st.req'd} := A_{sf} + \sqrt{A_{sv}^2 + A_l^2} = 12.161\ in^2$$

Provided reinforcement $A_{st} = 11.43\ in^2$

Demand/capacity ratio $DCR_{long.reinf} := \frac{A_{st.req'd}}{A_{st}} = 1.064$ $Check\left(\frac{A_{st.req'd}}{A_{st}} \le 1\right) = \text{"N.G."}$